AD NUMBER
AD326888
CLASSIFICATION CHANGES
TO: unclassified
FROM: confidential
LIMITATION CHANGES
TO: Approved for public release, distribution unlimited
FROM: Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; DEC 1961. Other requests shall be referred to Office of Naval Research, Arlington, VA 22203-1995.
AUTHORITY
29 Sep 1961, per doc markings; ONR ltr, 4 May 1977

TECHNICAL REPORT NO. 2

PENETRATION OF PULSED ELECTROMAGNETIC WAVES INTO CONDUCTING MEDIA

BY: L M Vallese
Dr. L. M. Vallese*
Senior Scientist

APPROVED BY: F A Muller
F. A. Muller
Laboratory Director

ITT FEDERAL LABORATORIES

a division of International Telephone and Telegraph Corporation
500 Washington Avenue
Nutley 10, New Jersey

*Paper presented at "ASW Electromagnetic Detection Seminar"
Office of Naval Research, Washington, D. C.
May 9 - 10, 1961
Contract NOnr 3358 (00)

4.35 ... 2.60

Introduction

The penetration of low frequency electromagnetic waves into conducting media such as the sea or the ground has received considerable attention for undersea and geodetic explorations as well as for communications. For such applications the field distribution within the medium as well as the distribution on the surface of separation with air are of interest. In the following the conducting medium is considered as uniform, unbounded in the horizontal direction, and bounded by a horizontal surface of given surface impedance at a depth d. The electromagnetic fields of interest have very low frequencies; examples are the fields at 10 kc to 20 kc used for navigational purposes, the fields produced by lightning or those obtained by man-made discharges[1]. At sufficient distance from the source these fields propagate as plane waves, guided by the plane surface of separation between air and conducting medium. More rigorously the propagation is determined also by the ionosphere, but in the following study the effect of the latter will be neglected.

In a previous paper [2] the author has analyzed the equivalent penetration depth of a pulsed d-c magnetic field wave into the conducting medium. A more complete analysis, including consideration of pulsed a-c magnetic field waves as well as of pulsed d-c or a-c electric field waves is given in the following. Part of this analysis was developed in a previous unpublished report [3], in which the application to the design of a system for submarine detection was developed. Such a system is described in detail in another paper of the Seminar [4].

1 - Distribution of Steady State Sinusoidal Fields

With reference to a rectangular system of coordinates with the $z = 0$ plane at the surface of separation between air and conducting medium (Figure 1), a plane wave with components E_x, E_z and H_y is considered. If 0, μ_o, ε_o and σ_1, μ_1, ε_1 are respectively conductivity, permeability and permittivity of the dielectric and of the conducting media, and if it is assumed that the media are unbounded except at the $z = 0$ plane, the field distribution of a low frequency plane wave propagating in the x direction is expressed as follows (5,6)

$z \leq 0$ air

$$H_y = H_o \exp[j\omega t - k_x x - k_z z]$$

$$E_z = -\frac{k_x}{j\omega\varepsilon_o} H_y \cong -\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_y \qquad (1)$$

$$E_x = -\sqrt{\frac{j\omega\mu_o\varepsilon_o}{\sigma_1}}\, H_y$$

$z \geq 0$ conducting medium

$$H_y = H_o \exp[j\omega t - k_x x - k_z' z]$$

$$E_z \cong -j\omega\frac{\sqrt{\mu_o\varepsilon_o}}{\sigma_1} H_y$$

$$E_x \cong -\sqrt{j\frac{\omega\mu_o}{\sigma_1}}\, H_y \qquad (2)$$

where

$$k_x \cong j\omega\sqrt{\mu_o\varepsilon_o}\sqrt{1 - j\frac{\omega\mu_1\varepsilon_o}{\sigma_1\mu_o}}$$

$$k_z \cong -j\omega\sqrt{\mu_o\varepsilon_o}\sqrt{j\omega\varepsilon_o/\sigma_1} \qquad (3)$$

$$k_z' = \sqrt{j\omega\mu_1\sigma_1}$$

The E_z component varies discontinuously from $z < 0$ to $z > 0$ decreasing in the ratio $1 : \omega\varepsilon_0/\sigma$. Furthermore, at $z < 0$, E_x is much smaller than E_z and leads it by $\pi/4$ radians; on the other hand at $z > 0$ E_x is much larger than E_z and lags it by $\pi/4$ radians. Without loss of generality the investigation may be confined to the case of a plane wave of components E_x, H_y normally incident on the $z = 0$ surface. The field distribution in the conducting medium is described by the wave equation

$$\left[\frac{\partial^2}{\partial z^2} - \sigma\mu\frac{\partial}{\partial t} - \varepsilon\mu\frac{\partial^2}{\partial t^2}\right]\begin{bmatrix}E\\H\end{bmatrix} = 0 \qquad (4)$$

where the last term, representing the displacement current, may be neglected in general.

Considering a plane wave of type

$$\begin{aligned} E_x &= E_0 \exp\left[j\omega t - k_z' z\right] \\ H_y &= H_0 \exp\left[j\omega t - k_z' z\right] \end{aligned} \qquad (5)$$

and assuming that the conducting medium is bounded by a horizontal plane of surface impedance Z_s at depth d, it is found that the ratio of the Fourier transforms $E_x(z,\omega)$ and $H_y(z,\omega)$ at $z = 0$ is expressed as follows

$$\frac{E_x(0,\omega)}{H_y(0,\omega)} = \frac{Z_s \operatorname{Cosh} k_z' d + Z_0 \operatorname{Sinh} k_z' d}{\operatorname{Cosh} k_z' d + \frac{Z_s}{Z_0} \operatorname{Sinh} k_z' d} \qquad (6)$$

where $Z_o = \sqrt{\frac{j\omega\mu_1}{\sigma_1}}$ is the wave impedance of the conducting medium. In particular, if $Z_s = 0$, i.e. if the boundary is a perfect conductor, the above ratio reduces to the expression

$$\frac{E_x[0,\omega]}{H_y[0,\omega]} = Z_0 \,\mathrm{Tanh}\, k_z' d = \sqrt{\frac{j\omega\mu_1}{\sigma_1}} \; \frac{\sinh\frac{2d}{\delta} + j\sin\frac{2d}{\delta}}{\cosh\frac{2d}{\delta} + \cos\frac{2d}{\delta}} \tag{7}$$

where $\delta = \sqrt{2/\omega\mu_1\sigma_1}$ is the so-called penetration depth. The magnitude and phase angle of the latter expression are plotted in Figure 2 as functions of the ratio d/δ; examination of Figure 2 shows that the significant range of variation of these quantities is approximately $0 \leq z \leq \delta$. In a practical application, assuming that the $Z_s = 0$ boundary conditions applies, the measurement of the magnitude and/or phase of (7) provides a means for the determination of the depth d at which the discontinuity is located. More generally, if the boundary has non zero surface impedance Z_s, the measurement may be used either to determine d, if Z_s is known, or Z_s if d is known.

It is of interest to note that, since $H_y(0,\omega)$ is related to $E_z(0,\omega)$ in air by the second of eq.s(1), the previous considerations are readily extended to the ratio $E_x[0,\omega]/E_z(0,\omega)$ where $E_z[0,\omega]$ is measured in air at the $Z = 0$ surface. With reference to eq. (7) one has

$$\frac{E_x[0,\omega]}{E_z[0,\omega]} = -\sqrt{\frac{\varepsilon_0}{\mu_0}} \sqrt{\frac{j\omega\mu_1}{\sigma_1}} \,\mathrm{Tanh}\, k_z' d \tag{8}$$

Generally the measurement of E is obtained by means of a dipole antenna, while that of H is obtained by means of a loop antenna.

A transmission line analogy of the previous analysis is readily set up, representing equation (6) as the input impedance of a line with series parameters $R = 0$, $L = \mu_1 N$ and shunt parameters $C = \varepsilon_1 N$, $G = \sigma_1 N$, where N is an arbitrary factor. Alternatively eq. (6) may be represented as the input admittance of a line with series parameters $R = \sigma_1 N$, $L = \varepsilon_1 N$ and shunt parameters $C = \mu_1 N$, $G = 0$ (Figure [illegible]). The use of these transmission lines permits the utilization of methods of measurement by substitution or by comparison, thereby enhancing the sensitivity of the technique. For example, one can vary the load impedance Z_s and the distance d of the analog line until the input impedance becomes equal to that measured at the air-ground or air-sea interface. Similarly, other methods of measurement may readily be derived.

II - Distribution of Transient Fields

The analysis of the distribution of transient fields may be performed with a similar formalism, using Laplace transforms of the E and H fields. Assuming that the said fields are zero at $t < 0$, the wave equation is written as follows

$$\left[\frac{d^2}{dz^2} - \sigma_1 \mu_1 s - \varepsilon_1 \mu_1 s^2\right] \begin{bmatrix} E\,[z,s] \\ H\,[z,s] \end{bmatrix} = 0 \tag{9}$$

where E (z,s) and H (z,s) are the Laplace transforms of E (z,t) and H (z,t) respectively. The general solution of the wave equation is expressed as a linear combination of two exponentials; if the presence of the boundary plane at depth d with surface impedance Z_s is postulated, the surface impedance at $z = 0$ is written as follows

$$\frac{E_x(0,s)}{H_y(0,s)} = \frac{Z_s \operatorname{Cosh} k_2' d + Z_0 \operatorname{Sinh} k_2' d}{\operatorname{Cosh} k_2' d + \frac{Z_s}{Z_0} \operatorname{Sinh} k_2' d} \qquad (10)$$

where $Z_0 = \sqrt{s\mu_1/\sigma_1}$ and $k_2' = \sqrt{s\,\mu_1\,\sigma_1}$.

In particular, if $Z_s = 0$, the latter expression is written more simply as follows

$$\frac{E_x(0,s)}{H_y(0,s)} = \sqrt{\frac{s\mu_1}{\sigma_1}} \operatorname{Tanh} \sqrt{s\mu_1\sigma_1}\, d \qquad (11)$$

Eq. (10) represents a boundary condition of the problem, relating the Laplace transforms of E and H at $z = 0$. In particular if $Z_s = 0$, it reduces the eq. (11), and if $d = \infty$, it reduces to the familiar

$$\frac{E_x(0,s)}{H_y(0,s)} = \sqrt{\frac{s\mu_1}{\sigma_1}} \qquad (12)$$

The latter case is generally more easily handled with the available Laplace transform tables. For example, in Table I the field distributions for the cases of a step d-c electric field, a step d-c magnetic field, a step a-c electric field and a step d-c magnetic field are given in complex form as well as in the time domain. Step electric fields may be generated at the $z = 0$ surface using a source of very low internal impedance, while step magnetic fields may be generated using sources of very high internal impedance. For example, fields generated

Table I - Field Distributions at Depth z for $d = \infty$

Applied Field	Field at Depth z
$E_x(0,s) = E_{x0}/s$	$E_x(z,s) = E_{x0}\frac{1}{s}\exp\left(-\sqrt{\sigma_1\mu_1 s}\, z\right)$
$E_x(0,t) = E_{x0}U(t)$	$E_x(z,t) = E_{x0}\,\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{\mu_1\sigma_1}{t}}\right)U(t)$
$H_y(0,t) = 2E_{x0}\sqrt{\frac{\sigma_1}{\mu_1\pi t}}$	$H_y(z,t) = E_{x0}\sqrt{\frac{\sigma_1}{\mu_1}}\left[2\sqrt{\frac{t}{\pi}}\exp\left(-\frac{\sigma_1\mu_1 z^2}{4t}\right) - z\sqrt{\mu_1\sigma_1}\,\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{\mu_1\sigma_1}{t}}\right)\right]U(t)$
$H_y(0,s) = H_{y0}/s$	$H_y(z,s) = H_{y0}\frac{1}{s}\exp\left[-\sqrt{\mu_1\sigma_1 s}\, z\right]$
$H_y(0,t) = H_{y0}U(t)$	$H_y(z,t) = H_{y0}\,\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{\mu_1\sigma_1}{t}}\right)U(t)$
$E_x(0,t) = \frac{H_{y0}\sqrt{\mu_1}\,U(t)}{\sqrt{\sigma_1\pi t}}$	$E_x(z,t) = H_{y0}\sqrt{\frac{\mu_1}{\sigma_1\pi t}}\exp\left[-\frac{z^2\mu_1\sigma_1}{4t}\right]U(t)$
$E_x(0,s) = \frac{E_{x0}\omega}{s^2+\omega^2}$	$E_x(z,s) = \frac{E_{x0}\omega}{s^2+\omega^2}\exp\left[-\sqrt{\sigma_1\mu_1 s}\, z\right]$
$E_x(0,t) = E_{x0}\sin\omega t\, U(t)$	$E_x(z,t) = \frac{z\sqrt{\mu_1\sigma_1\omega}}{2\sqrt{\pi}}E_{x0}\int_0^{\omega t}\frac{\sin\omega(t-\tau)\,\varepsilon^{-\frac{\mu_1\sigma_1 z^2\omega}{4\omega\tau}}}{(\omega\tau)^{3/2}}\,d\omega\tau$
$H_y(0,s) = \sqrt{\frac{\sigma_1}{\mu_1 s}}\frac{E_{x0}\omega}{s^2+\omega^2}$	$H_y(z,t) = \sqrt{\frac{\sigma_1}{\omega\mu_1\pi}}E_{x0}\int_0^{\omega t}\frac{\sin\omega(t-\tau)\,\varepsilon^{-\frac{\mu_1\sigma_1 z^2\omega}{4\omega\tau}}}{(\omega\tau)^{1/2}}\,d\omega\tau$
$H_y(0,s) = \frac{H_{y0}\omega}{s^2+\omega^2}$	$H_y(z,s) = H_{y0}\frac{\omega}{s^2+\omega^2}\exp\left[-z\sqrt{\sigma_1\mu_1 s}\right]$
$H_y(0,t) = H_{y0}\sin\omega t\, U(t)$	$H_y(z,t) = H_{y0}\frac{z}{2}\sqrt{\frac{\mu_1\sigma_1\omega}{\pi}}\int_0^{\omega t}\frac{\sin\omega(t-\tau)\,\varepsilon^{-\frac{\mu_1\sigma_1 z^2\omega}{4\omega\tau}}}{(\omega\tau)^{3/2}}\,d\omega\tau$
$E_x(0,s) = \sqrt{\frac{\mu_1 s}{\sigma_1}}\frac{H_{y0}\omega}{s^2+\omega^2}$	$E_x(z,t) = H_{y0}\sqrt{\frac{\mu_1\omega}{\sigma_1\pi}}\int_0^{\omega t}\frac{\sin\omega(t-\tau)\left[z^2\mu_1\sigma_1\omega - 2\omega\tau\right]\varepsilon^{-\frac{\mu_1\sigma_1 z^2\omega}{4\omega\tau}}}{4(\omega\tau)^{5/2}}\,d\omega\tau$

by sources within the atmosphere may be considered as step magnetic fields because the wave impedance of air is much larger that that of ground or sea. On the other hand step electric fields may be applied at $z = 0$ by generating a step potential difference across two parallel electrodes located at the $z = 0$ surface; similarly, step magnetic fields may be applied at $z = 0$ by generating a step current in a loop with axis on the $z = 0$ surface.

Plots of the field distributions for the cases of step d-c electric fields and step d-c magnetic fields are given in normalized form in Figures 4 and 5 using as independent variable $t^* = 4t/\mu_1\sigma_1 z^2$. For a given value of z, except $z = 0$, the abscissas are proportional to the real time. For completion, the plots of Figures 4 and 5 include also the distributions of pulses of normalized duration $T_o^* = 4$, i.e. of real duration $T_o = \mu_1\sigma_1 z^2$ where z is a given value. It is of interest to note that, when a rectangular pulse d-c electric field is applied, the corresponding electric field is an alternating pulse. The zero cross-over point of the latter pulse is a function of the distance z and of the duration T_o. Indicating with T' the time of zero cross-over, one finds the following relationship

$$z^2 = \frac{2T_o}{\sigma_1\mu_1} \frac{\ln\left[1 - T_o/T'\right]}{\frac{T_o}{T' - T_o} - \frac{T_o}{T'}} \tag{13}$$

The latter result could be of interest for application to techniques of ranging or of communications between two points within the dissipative medium. In fact, if a plane wave consisting of a rectangular pulse d-c magnetic field is generated at one point and is received at the other at distance L, the measurement of T' provides information about L if T_o is known, or about T_o if L is known. In addition radar like techniques of ranging may be applied. A numerical example with medium corresponding to average sea water ($\mu_1 = \mu_o$, $\sigma = 4$ ℧/m) and with $H_o = 1$ A/m, L = 100 m, $T_o = 25$ msec has been computed and is plotted in Figure 6. It is seen that T' = 40.5 msec and that the peak values of the E and H pulses are respectively 1.2 mV/m and 0.33 A/m at such distance.

Plots of the field distributions for the cases of step a-c electric fields and step a-c magnetic fields cannot be given in normalized form, because the fields are represented by means of integral expressions, which cannot be computed in terms of known tabulated functions. For this reason a numerical example has been computed, assuming that $z/\delta = z\sqrt{\omega\mu\sigma}/\sqrt{2} = \pi/2$. The distributions of the electric field component E_x versus ωt at $z = \pi\delta/2$ are shown in Figures 7 and 8 respectively for the case of an abruptly applied sine wave $E_x(0, t)$ and of an abruptly applied sine wave $H_y(0, t)$.

More generally, by application of the boundary condition (10) (or in particular (11)), it is possible to evaluate the distribution of transient fields at the z = 0 surface as determined by the presence of a boundary of surface impedance Z_s at depth z. However, the analysis in general is very cumbersome and does not lead to sufficiently simple expressions. If the transmission line analogy previously described is utilized, the distributions may be obtained in analog form, and may be applied directly for practical methods of measurement based on substitution or on comparison procedures, similar to those described for the case of steady state fields.

III - Conclusions

In the previous analysis the distributions of steady state and of transient electromagnetic plane waves at the surface of separation between air and ground (or sea) have been computed. These distributions are important for such applications as undersea and geodetic explorations from points located outside the conducting media; the methods of measurement may be simplified by use of a transmission line analogy, which permits the use of comparison or of substitution methods.

In addition, it has been shown that, if a rectangular pulse d-c magnetic plane wave is transmitted from a point within the conducting medium and received at another point of the same, the system may be used for purposes of ranging or for purposes of communication between transmitting and receiving stations.

Acknowledgement

The support of the Office of Naval Research, and, in particular, that of Dr. A. Shostak, Head of the Electronics Branch, is gratefully acknowledged.

REFERENCES

1. M. J. Larsen - "Lightning Studies", Seminar on Electromagnetic Detection in Antisubmarine Warfare - O. N. R., Washington 25, D. C. May 9 - 10, 1961.

2. L. M. Vallese - "Diffusion of Pulsed Currents into Conductors", Journal of Applied Physics - Vol.25, No.2, p 225-228, 1954.

3. L. A. deRosa and L. M. Vallese - "Contributions to the Study of Field Perturbations by Submarines", ITT Laboratories, Nutley, N. J. Document 64228, October 1959.

4. F. Modavis - "Project Dinah" - "Seminar on Electromagnetic Detection in Antisubmarine Warfare" - O. N. R., Washington 25, D.C., May 9 - 10, 1961.

5. J. A. Stratton - "Electromagnetic Theory" Chapter IX, 1941 - McGraw Hill Book Company.

6. E. C. Jordan - "Electromagnetic Waves and Radiating Systems", Chapter 7 Prentice Hall Co., 1950.

LIST OF ILLUSTRATIONS

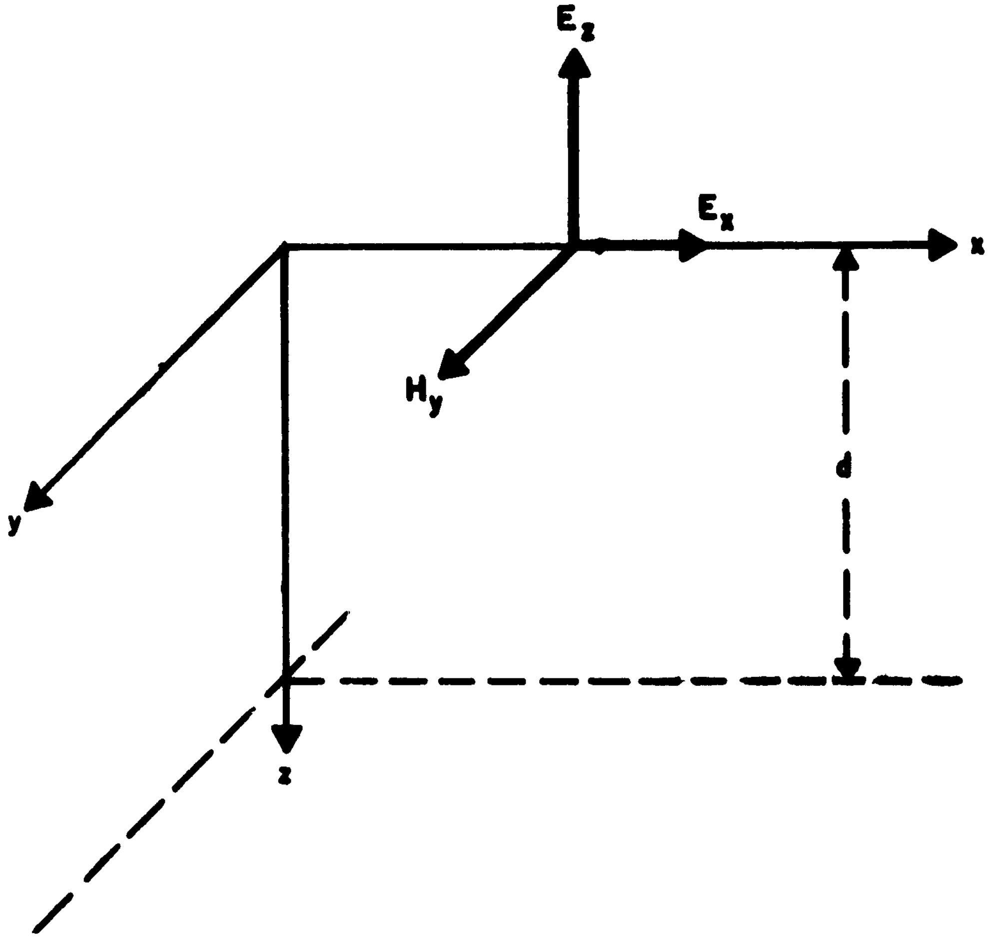


FIGURE 1
RECTANGULAR COORDINATE SYSTEM

A2293033

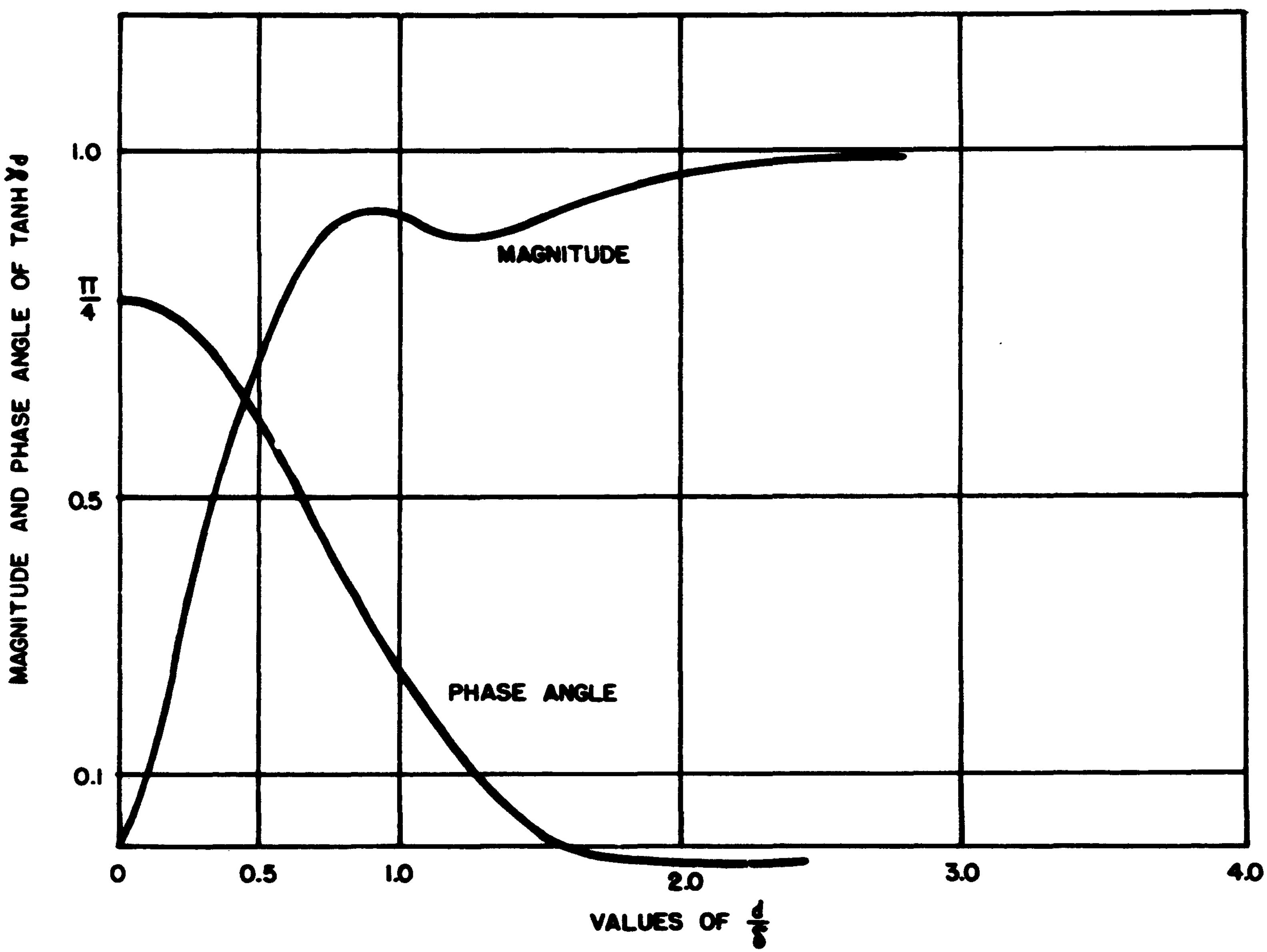
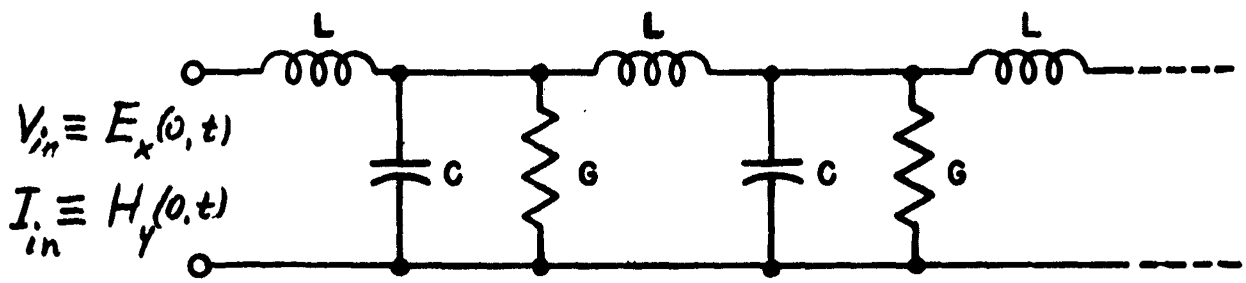
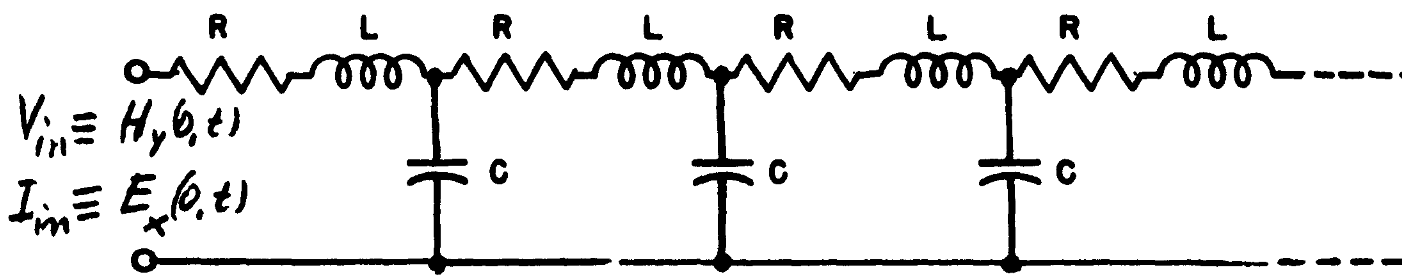


FIGURE 2 PLOT OF TANH γd



(A)



(B)

FIGURE 3: ANALOG TRANSMISSION LINES

FIGURE 4: WAVEFORM OF ELECTRIC AND MAGNETIC FIELDS AT A GIVEN DISTANCE Z ASSUMING AN INITIAL E PERTURBANCE

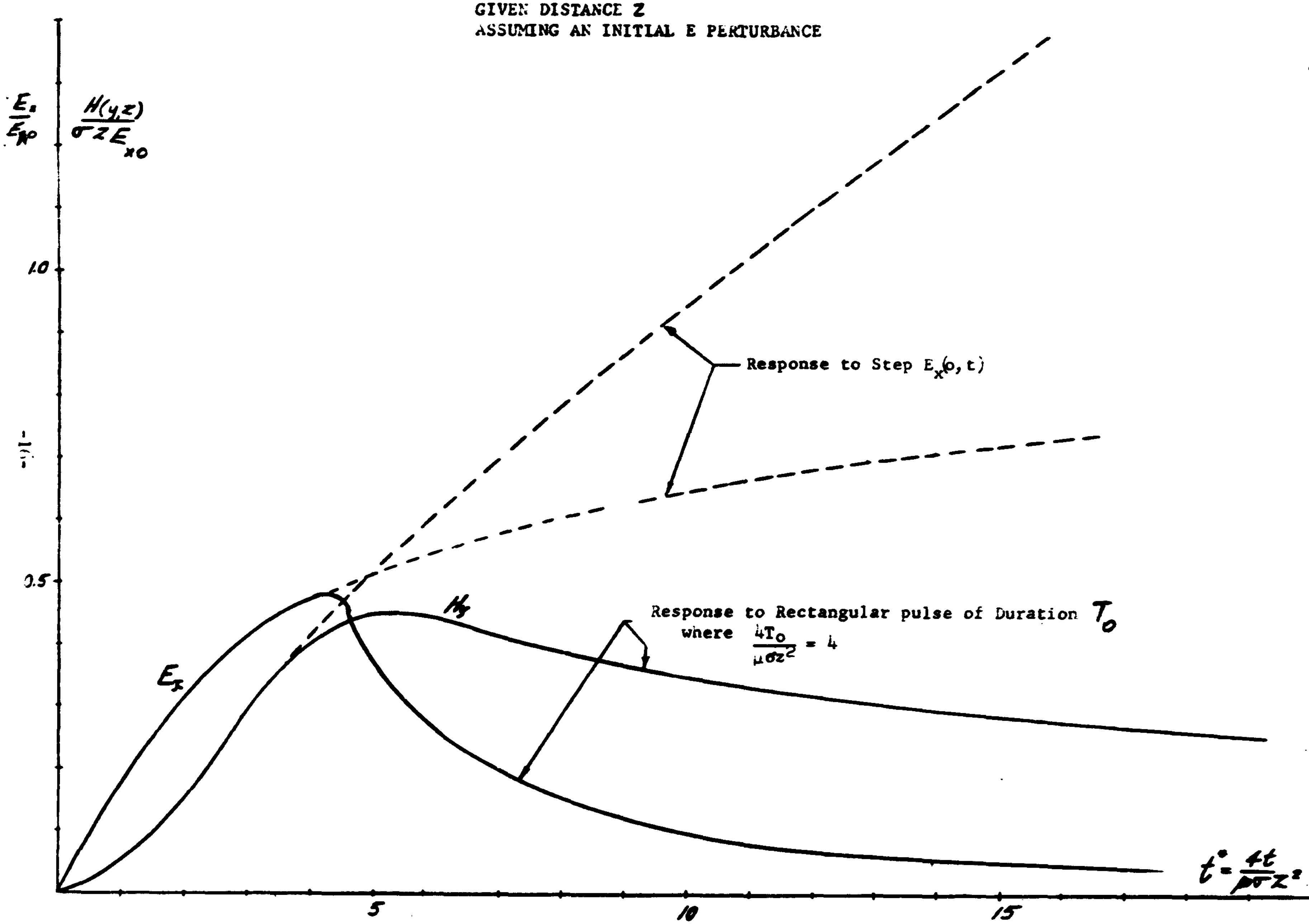


FIGURE 4

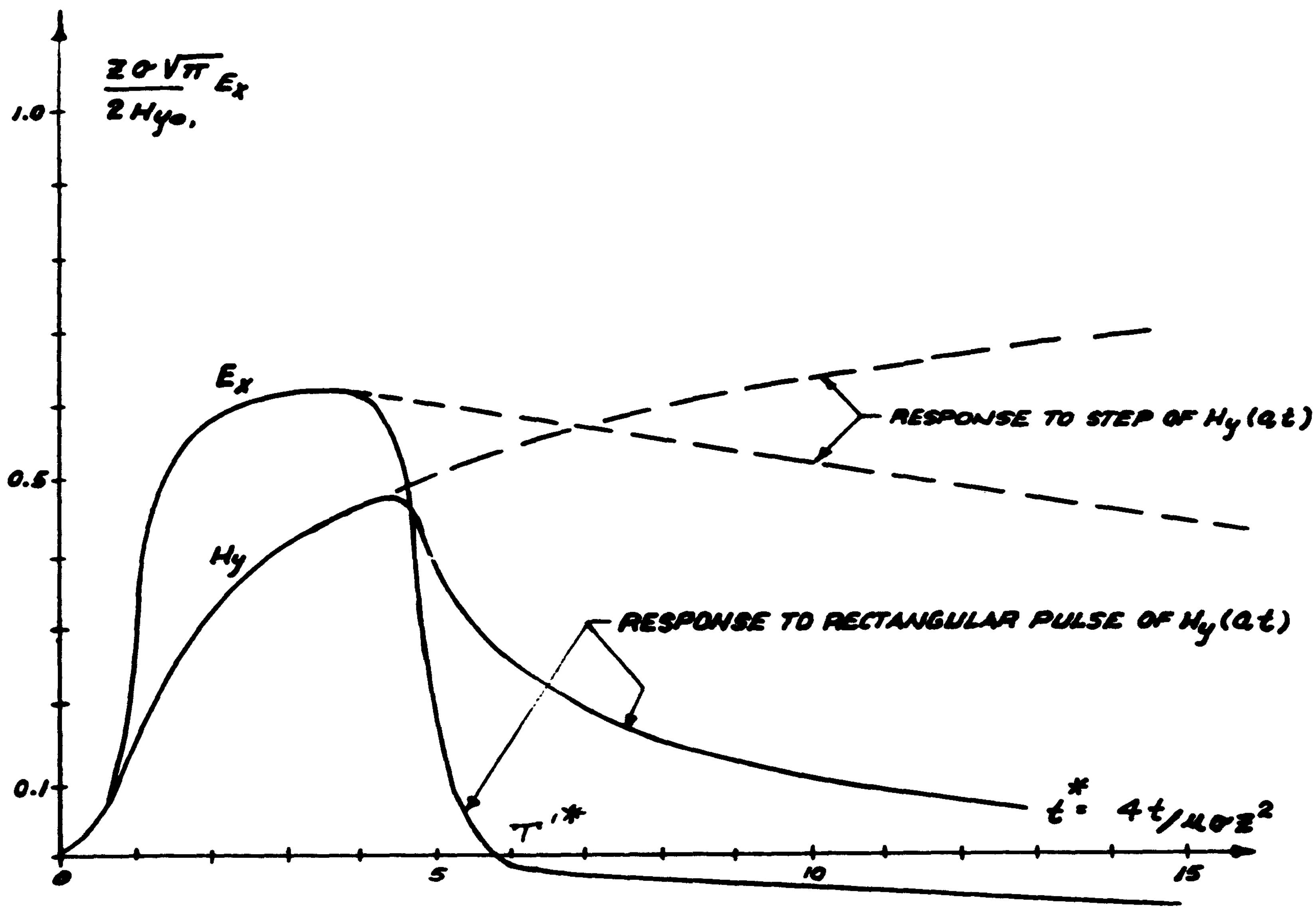


FIGURE 5: DIFFUSION OF A STEP OR PULSE OF $H_y(0,t)$

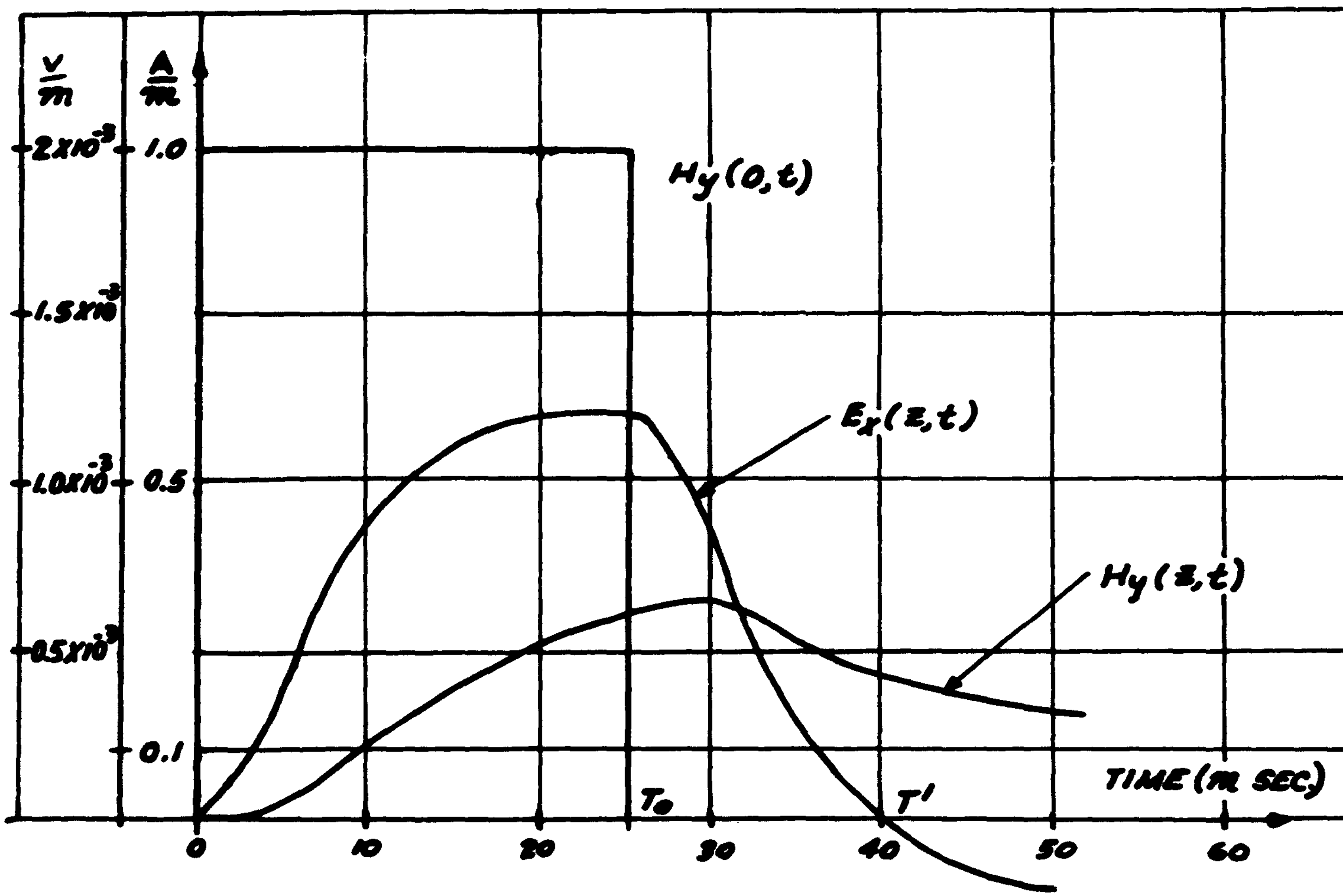


FIGURE 6: FIELD DISTRIBUTION OF A PULSE OF $H_y(0,t)$ AT 100m DEPTH

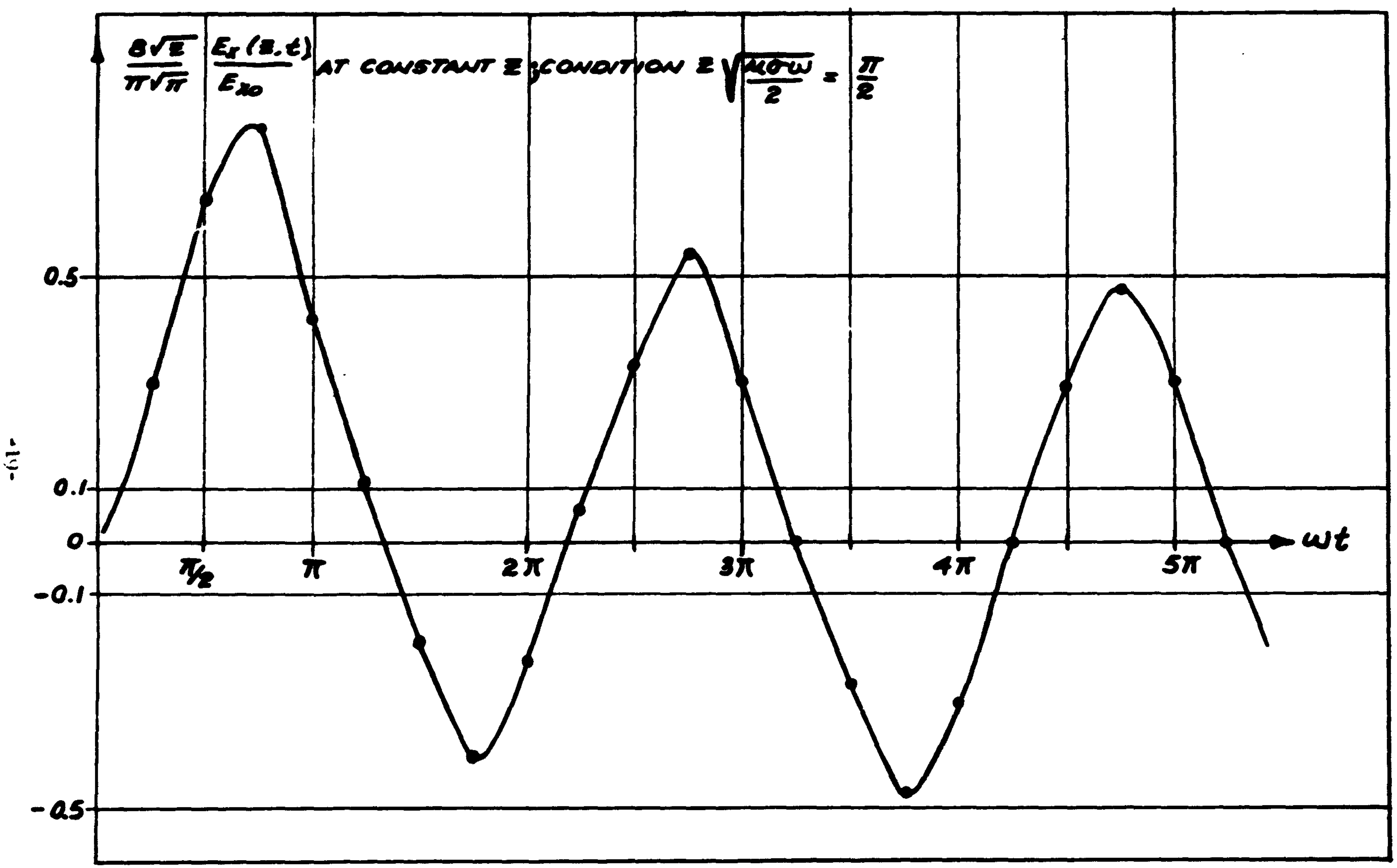


FIGURE 7: DIFFUSION OF AN ABRUPTLY APPLIED SINE WAVE $E_x(0,t)$

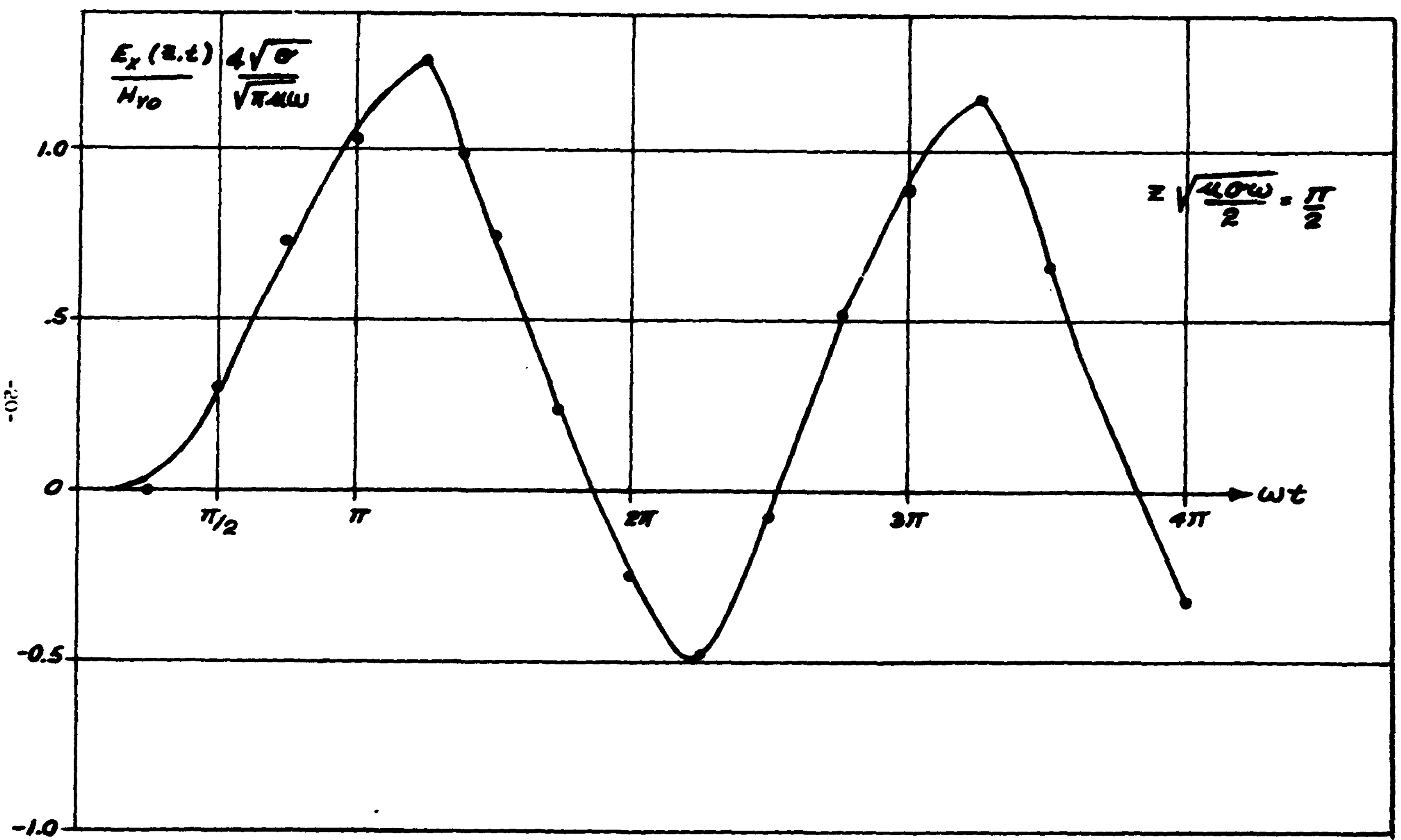


FIGURE 8: DIFFUSION OF AN ABRUPTLY APPLIED SINE WAVE $H_y(0,t)$